Morningstar Risk-Adjusted Return

The Morningstar Risk-Adjusted Return (MRAR) measure has the following characteristics:

- no particular distribution of excess returns is assumed
- risk is penalized in all cases
- the theoretical foundation is acceptable to sophisticated investors and investment analysts.

MRAR is motivated by expected utility theory, according to which an investor ranks alternative portfolios using the mathematical expectation of a function (called the utility function) of the ending value of each portfolio.

Let W be the ending value of a portfolio being considered and u(.) be the investor's utility function. The expected utility of the portfolio is E[u(W)].

To be meaningful, the utility function must satisfy two conditions. First, it must always be positively sloped; i.e., $u'(.)>0$. That is, more expected wealth is always better than less expected wealth. Second it must imply risk aversion, i.e., that the investor prefers a riskless portfolio with a known end-of-period value to a risky portfolio that is expected, but not certain, to have the same end-of-period value. This means:

[5] $$u(E[W])>E[u(W)]$$

From probability theory, it follows that this can be true only if u(.) is everywhere a concave function; i.e. $u''(.)<0$.

The shape of the utility function describes the investor's attitude toward risk. The degree of risk aversion can be measured by the coefficient of relative risk aversion, RRA:

[6] $$RRA(W)=-\frac{Wu''(W)}{u'(W)}$$

A form of the utility function that is especially useful in portfolio theory is constant relative risk aversion. RRA(.) being a constant implies that u(.) can be written as:

$$[7] \qquad u(W)=\begin{cases} -\dfrac{W^{-\gamma}}{\gamma} & \gamma>-1,\ \gamma\neq 0 \\ \ln(W) & \gamma=0 \end{cases}$$

where γ is a parameter that describes the degree of risk aversion, specifically, RRA(.) = $\gamma+1$.

Constant relative risk aversion also implies that the investor's beginning-of-period wealth has no effect on the ranking of portfolios. To see this, let:

W_0	=	beginning of period wealth
TR	=	total return on the portfolio being evaluated so that $W = W_0 (1+TR)$

Hence:

$$[8] \qquad u(W_0(1+TR))=\begin{cases} W_0^{-\gamma}\, u(1+TR) & \gamma>-1,\ \gamma\neq 0 \\ \ln(W_0)+u(1+TR) & \gamma=0 \end{cases}$$

The value of W_0 does not affect the curvature of utility as a function of TR, and so it does not affect how the investor ranks portfolios.

Instead of holding a risky portfolio, the investor could buy a risk-free asset. Let R_b be the return on the risk-free asset. In comparing risky portfolios to the risk-free asset, we assume that the investor initially has all wealth invested in the risk-free asset and beginning-of-period wealth is such that end-of-period wealth, so invested, will be $1.

Hence:

[9] $$W_0 = \frac{1}{1+R_b}$$

and

[10] $$u(W_0(1+TR)) = u\left(\frac{1+TR}{1+R_b}\right) = u(1+r_G) = \begin{cases} -\frac{(1+r_G)^{-\gamma}}{\gamma} & \gamma > -1,\ \gamma \neq 0 \\ \ln(1+r_G) & \gamma = 0 \end{cases}$$

where

[11] $$r_G = \text{ the geometric excess return } = \frac{1+TR}{1+R_b} - 1$$

The certainty equivalent geometric excess return of a risky investment is the guaranteed geometric excess return that the investor would accept as a substitute for the uncertain geometric excess return of that investment. Letting $r_G^{CE}(\gamma)$ denote the certainty equivalent geometric excess return for a given value of γ, this means that:

[12] $$u(1+r_G^{CE}(\gamma)) = E[u(1+r_G)]$$

Hence:

[13] $$r_G^{CE} = \begin{cases} \left(E[(1+r_G)^{-\gamma}]\right)^{-\frac{1}{\gamma}} & \gamma > -1,\ \gamma \neq 0 \\ e^{E[\ln(1+r_G)]} & \gamma = 0 \end{cases} - 1$$

MRAR(γ) is defined as the annualized value of r_G^{CE} using the time series average of $(1+r_G)^{-\gamma}$ as an estimate of $E[(1+r_G)^{-\gamma}]$. With $\gamma\neq 0$, we have:

$$\text{MRAR}(\gamma)=\left[\frac{1}{T}\sum_{t=1}^{T}(1+r_{Gt})^{-\gamma}\right]^{-\frac{12}{\gamma}}-1 \qquad [14]$$

where

r_{Gt}	=	the geometric excess return in month t $=\frac{1+TR_t}{1+R_{bt}}-1$
R_{bt}	=	return on risk-free asset in month t

When $\gamma=0$, MRAR is the annualized geometric mean of r_G:

$$\text{MRAR}(0)=\left[\prod_{t=1}^{T}(1+r_{Gt})\right]^{\frac{12}{T}}-1 \qquad [15]$$

A rating system based solely on performance would rank funds on their geometric mean return, or equivalently, MRAR(0). A rating system that provides a heavier penalty for risk requires that $\gamma>0$.

Morningstar's U.S. fund analysts have concluded that $\gamma=2$ results in fund rankings that are consistent with the risk tolerances of typical retail investors.
Hence, Morningstar uses a γ equal to 2 in the calculation of its star ratings.

Because MRAR is expressed as an annualized return, it can be decomposed into a return component, MRAR(0), and a risk component, MRAR(0)–MRAR(2).